X. THROUGHPUT ANALYSIS OF S/R SHUTTLE SYSTEMS

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Abstract

Shuttle systems are used in high performance automated storage/retrieval unit load systems. Each storage level is serviced by one transfer car travelling in dual command operation. One buffer slot is located at both ends of each level. This decouples horizontal travel from vertical input/output moves, which in this case requires two independent vertical reciprocating lifts at each end of the aisle. Other systems work with only one lift used in lower throughput applications. The content of this paper is treated in the following sections starting with a problem definition and a literature survey. This is followed by a detailed functional description of the system investigated here. A predictive model with analytical equations is derived for simplified calculations and a comparison with simulation results. A summary, conclusions and an outlook finalize the paper.

1 Introduction and problem definition

During the past years a number of new automated storage and retrieval systems based on individual autonomous vehicles - also called shuttle systems - have been introduced to the European logistics market [1]. As compared to the conventional AS/R systems they offer greater operational flexibility in varying demand conditions. Several arrangements for the main components consisting of racks, lifts, buffers and vehicles are possible. A number of research papers has been published in the past to investigate standard configurations [2, 7, 8, 9, 10].

In this paper a special configuration of devices to be described later in more detail is investigated in its main performance measures throughput and cycle time. Analytical equations are presented to calculate throughput as the number of transactions per unit time and cycle time as the function of rack geometry (length, height) and kinematic data (velocities, accelerations, transfer times) from the beginning until the end of a transaction. The results of the analytical calculations verified with the simulation package ARENA. Furthermore a system comparison of the shuttle system with conventional SR systems is presented.

The following research questions are treated in detail:

> ➢ Calculation of cycle times and throughput depending on rack size
> ➢ are deadlock situations with blocking possible and by which operational rules can they be prevented
> ➢ what is the effect on throughput and cycle times if shuttles perform single command (SC) operations instead of double command (DC) operations
> ➢ which improvement of throughput of a shuttle system can be expected compared to a conventional S/R system

Further research will be extended to the influence of the dwell point position, the I/O location and the number of buffers on the system performance.

2 Literature survey

As mentioned a number of research papers [7, 8, 9, 10] have been published to investigate performance measures of the shuttle technology in the last years. A major research has been published by Roy (2011) in his dissertation showing the basic system design in Fig.1.
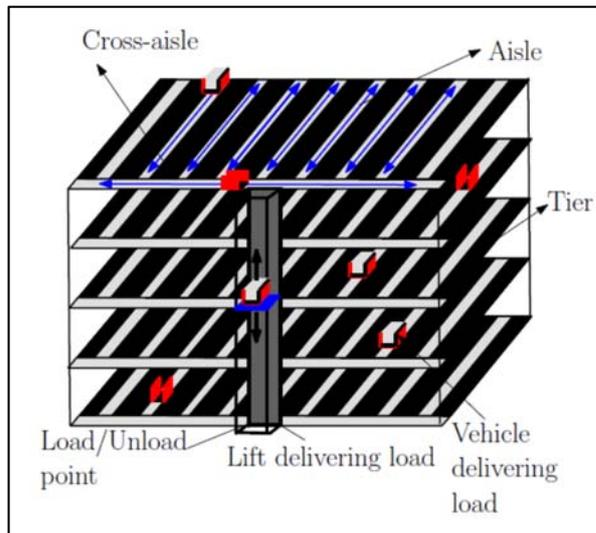
Figure 1: Autonomous vehicle based storage and retrieval (AVS/R) systems as described in [2]

A short functional description of the system investigated by Roy exhibits the following features:

> ➢ The shuttles operate on each tier in X- and Y- direction

> ➢ Several shuttles operate on each tier simultaneously

> ➢ Shuttles can change between tiers

> ➢ Loads can enter or leave the system only by means of a shuttle at the I/O point

As will be shown in the next section these operational specifications do not apply to the system investigated here.

Standard shuttle system configurations have also been treated in VDI 2692 (2013) [3].The cycle time equations presented there allow only a separate calculation of shuttle and lift equations. VDI 2692 always assumes the lift to act as bottleneck and therefore considers only the lift cycle time to be relevant for the system performance.

Due to interference processes between shuttle, lift and waiting times transfer processes between the devices have to be considered

3 Detailed description of the shuttle system

3.1 System description (Figure 2, 3)

The shuttle system contains four devices:

Rack
The rack contains the storage locations for the unit loads and horizontal guidance rails for the shuttles. Average rack dimensions have lengths varying between 10 and 150 meters and heights between 2 and 30 meters. Storage locations can be single, double, triple and quadruple deep. Here only single deep locations are treated.

Shuttles
The shuttles contain transfer devices operating in orthogonal direction to the main guidance rails. After the load transfer on the shuttle it travels along the horizontal x-axis from the I/O point to the required storage location. Reverse operation takes place in case of a retrieval. Only one shuttle travels on each tier.

Lift
Vertical transportation along the z-Axis is enabled by two separate lifts, one for the input and one for the output operation. The input lift moves the load from the I/O point at $z = 0$ to the input buffer at the required tier. The output lift operates in the opposite direction.

Buffers
Each tier contains one input buffer and one output buffer arranged opposite of each other. When retrieving a load the shuttle transfers it from the vehicle platform to the buffer place, where from it will be removed by the output lift at a later point of time. Thus the buffer enables a decoupling of shuttle and lift operations. As the shuttle has no waiting time at the output lift it can service a different function and improve performance.

Figure 2: Shuttle system (OSR Shuttle by courtesy of KNAPP)

Figure 3 exhibits the geometric arrangement and the system layout of the shuttle system with buffer locations at the end of the main aisle. Also a double lift system is possible when lift locations are arranged at both ends of the aisle. But not investigated in this paper.
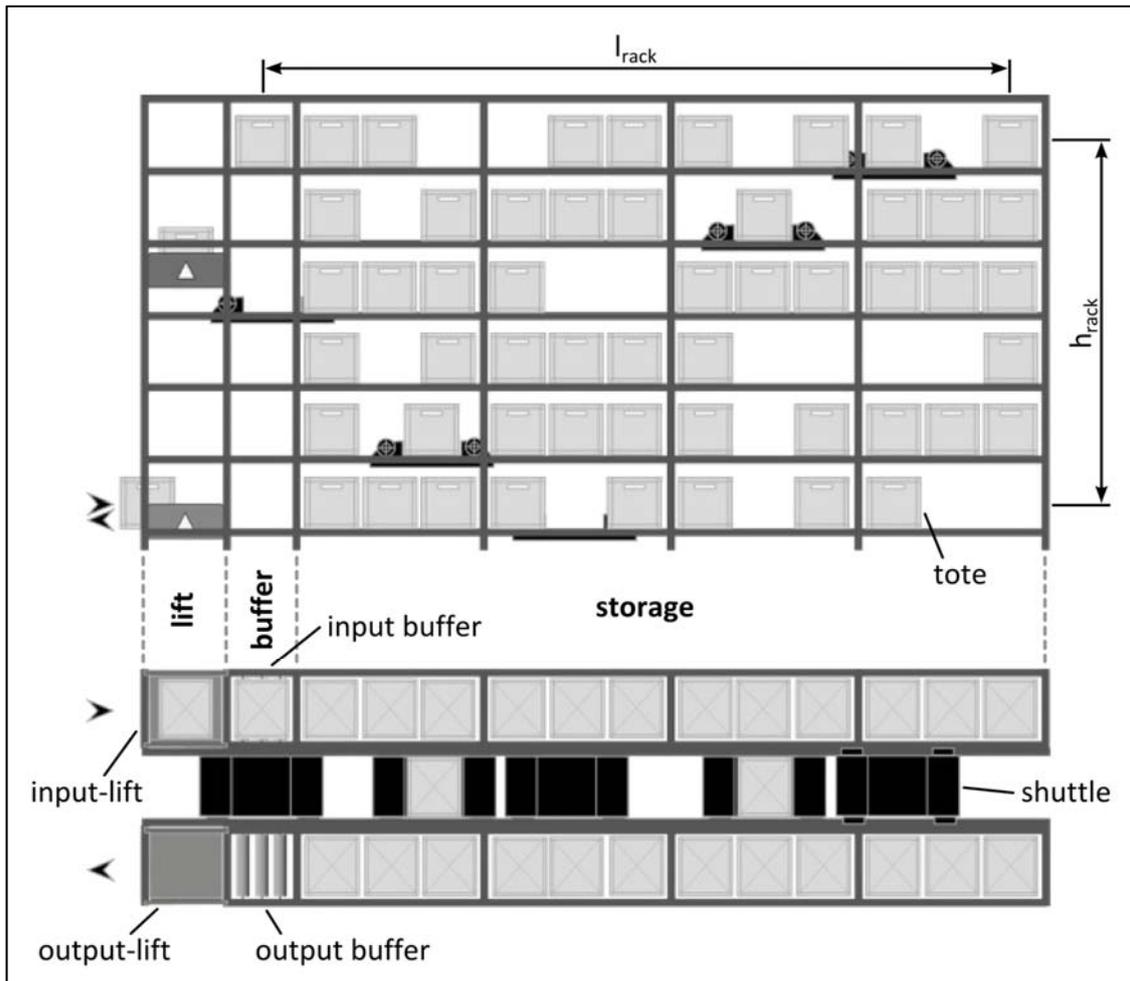
Figure 3: Shuttle system

3.2 Functional description

3.2.1 Input process

The input process describes the movement of loads from the I/O point to the storage position in the rack. A load to be stored waits in the queue at the I/O point until a vertical motion of the lift transports the load to the target level and transfers it to the input buffer position. Here it will wait until it can be transferred to the shuttle, which transports the load horizontally and transfers it to the final storage position (Figure 4).
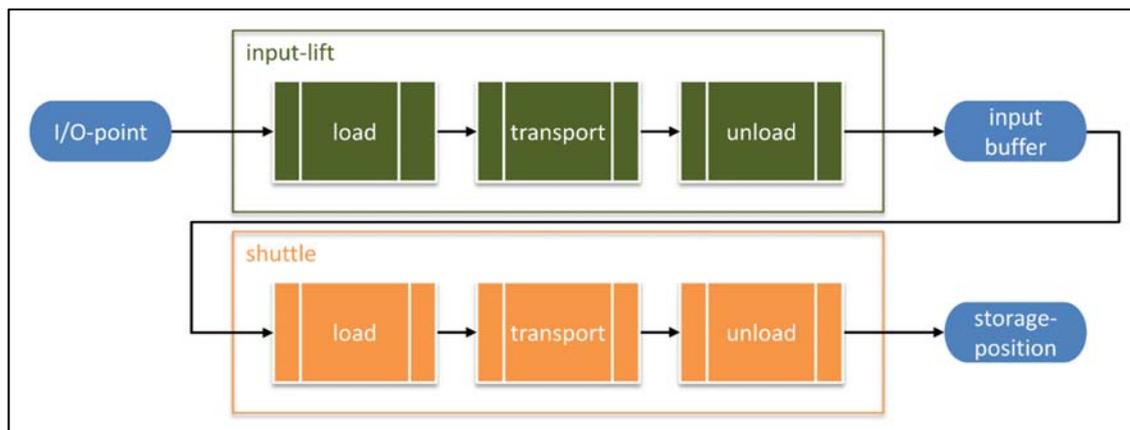
Figure 4: Input process

3.2.2 Output process

The output process describes the movement of loads from the retrieval location to the I/O position in front of the rack. A load to be retrieved waits in the order queue at the retrieval location until a horizontal motion of the shuttle transports the load to the output buffer position. After a possible waiting time to empty the buffer a transfer mechanism moves the load to the output buffer position. Here it will wait until it can be transferred to the vertical lift, which transports it to the final I/O position (Figure 5).

The control rules operate the lifts in single commands and the shuttles in dual commands. This means that a shuttle after transporting a load to the storage position moves in empty travel to the next retrieval position, wherefrom it transports the retrieval load to output buffer and removes it by a lift operation.
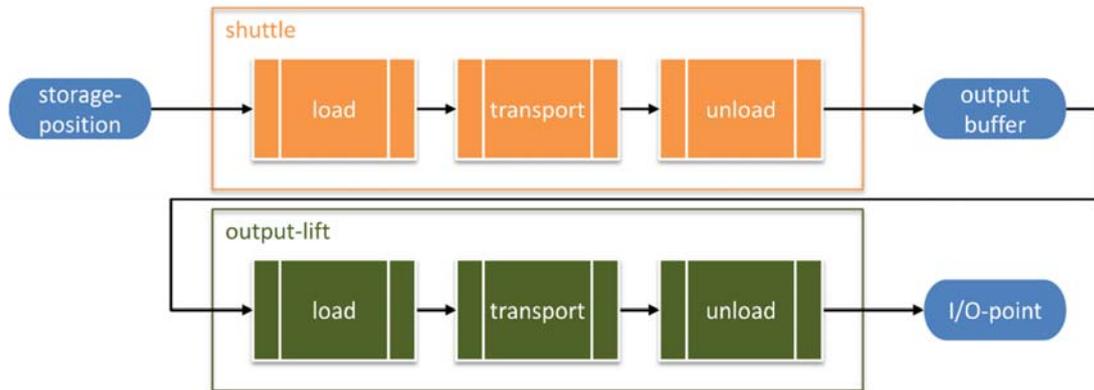
Figure 5: Output process

4 Analytical calculation of cycle times and throughput

4.1 Cycle times and throughput with lifts performing single cycles and shuttles performing double cycles

As stated before the input and the output lifts operate in single cycles and the shuttles in dual cycles. These two operations are executed at the same time. For the calculation of the cycle time of the shuttle system the maximum of lift cycle time and shuttle cycle time is dominant.

Additionally to lift cycle times and shuttle cycle times there may occur waiting times. For example in the output process it is possible, that the output lift has to wait for totes brought by a shuttle. It is also possible that too many totes are brought to the output lift and have to wait to be served.

So the expected value of the cycle time of the complete shuttle system $E(t_{doublecycle})$ is:

$$E\left(t_{doublecycle}\right) := \max\left(t_{lift}, \frac{t_{shuttle}}{n}\right) + t_{wait}$$

The expected value of the cycle time of the lift is:

$$E\left(t_{lift}\right) := \frac{h_{rack}}{v_{lift}} + 2 \cdot \frac{v_{lift}}{a_{lift}} + t_{load\_lift} + t_{unload\_lift}$$

The expected value of the cycle time of the shuttle is:

$$E\left(t_{shuttle}\right) := \frac{4}{3} \cdot \frac{l_{rack}}{v_{shuttle}} + 3 \cdot \frac{v_{shuttle}}{a_{shuttle}} + 2 \cdot t_{load\_shuttle} + 2 \cdot t_{unload\_shuttle}$$

h_{rack} … height of the rack

l_{rack} … length of the rack

v_{lift} … velocity of the lifts

$v_{shuttle}$ … velocity of the shuttles

a_{lift} … acceleration of the lifts

$a_{shuttle}$ … acceleration of the shuttles

t_{loadl} … loading time of the lifts

$t_{unloadl}$ … unloading time of the lifts

t_{loads} … loading time of the shuttles

$t_{unloads}$ … unloading time of the shuttles

These equations were derived from the well-known formulas for the calculation of cycle times for AS/RS [4].

Additionally to the transportation times waiting times have to be considered:

➢ In the output process it is the waiting time of the shuttle until the output buffer becomes empty
➢ In the Input-Process it is the waiting time of the lift until the input buffer becomes empty

The waiting time can be calculated with the help of the queuing theory. Hereby different waiting models can be used. For example the G/G/1 model, M/G/1 model or the $E_k/E_k/1$

model. In section 5 it will be shown by simulation results that the M/G/1 model delivers the best approach.

If we consider the output process, the n shuttles in the rack describe the arrival process and the output lift describes the service process. If we look at the input process, the lift performs the arrival process and the n shuttles in the rack perform the service process.

So we can state for the output process:

$$\lambda_{output} := \frac{1}{\frac{E\left(t_{shuttle}\right)}{n}}$$

n … number of levels in the rack

$$\mu_{output} := \frac{1}{E\left(t_{lift}\right)}$$

$$\rho_{output} := \frac{\lambda_{output}}{\mu_{output}}$$

And for the input process

$$\lambda_{input} := \frac{1}{E\left(t_{lift}\right)}$$

$$\mu_{input} := \frac{1}{\frac{E\left(t_{shuttle}\right)}{n}}$$

$$\rho_{input} := \frac{\lambda_{input}}{\mu_{input}}$$

For the M/G/1 model the waiting times for the input and output process are [5]:

$$t_{wait\_output} := \frac{\rho_{output} \cdot t_{lift}}{1 - \rho_{output}} \cdot \frac{1 + v_s^2}{2}$$

$$t_{wait\_input} := \frac{\rho_{input} \cdot \dfrac{t_{shuttle}}{n}}{1 - \rho_{input}} \cdot \frac{1 + v_s^2}{2}$$

with the variational coefficient v_s for the service process.

$$Var(t_{shuttle}) := \frac{\left(2 \cdot \dfrac{l_{rack}}{v_{shuttle}}\right)^2}{18}$$

$$v_s := \frac{\sqrt{Var(t_{shuttle})}}{n \cdot t_{shuttle}}$$

The calculation relies on the following assumptions, which have been verified by simulation

1. The output cycle time of a system with an utilization ratio of ρ_{output} equals the input cycle time of the same system with $\rho_{input} = 1/\rho_{output}$. A numerical example proves the output cycle time of a system with $\rho_{output} = 2$ to equalize the calculation results with an input cycle time for $\rho_{input} = 0{,}5$.
2. Theoretically the waiting times would raise towards infinity in the region of $\rho_{output} \approx 1$ or $\rho_{input} \approx 1$. By simulation however it was found that the average waiting time with dual command operation never exceeded two seconds (Figures 6, 7): ➜ $t_{wait\_max} = 2$ sec
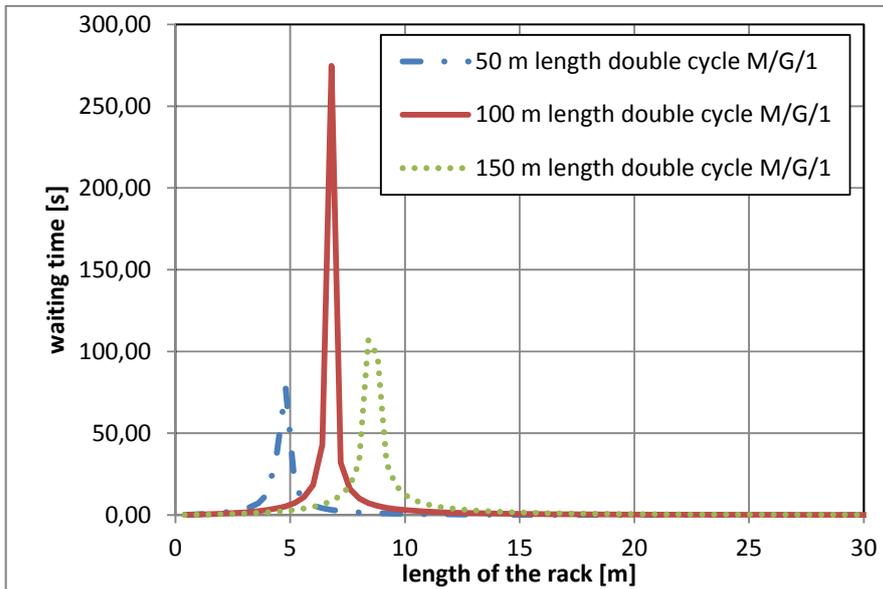
Figure 6: Waiting times
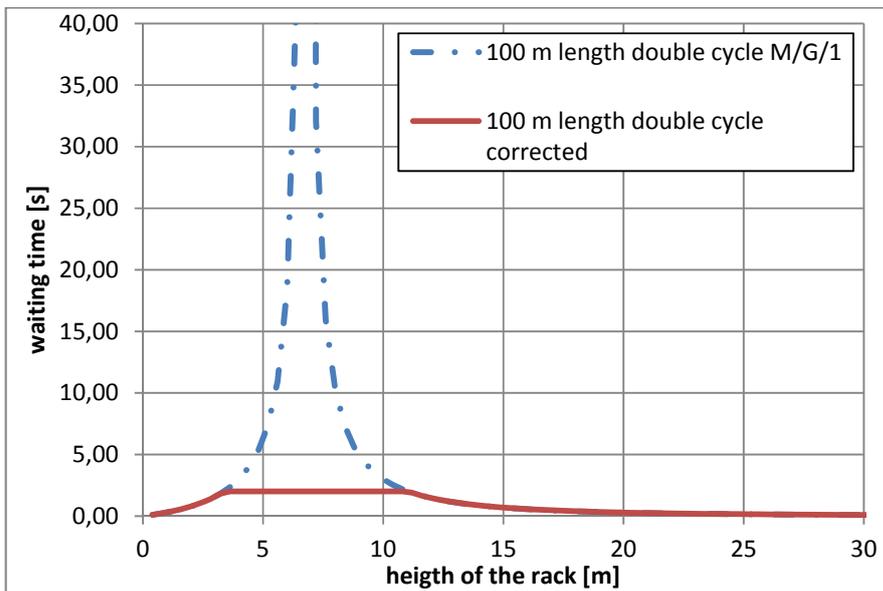
Figure 7: Waiting times for a 100 m rack with and without correction

For the calculation the following lift and shuttle parameters for lifts and shuttles where defined:

$v_{shuttle} = 2$ m/s	$v_{lift} = 5$ m/s	velocities of shuttles and lifts
$a_{shuttle} = 2$ m/s^2	$a_{lift} = 7$ m/s^2	acceleration of shuttles and Lifts
$t_{loads} = 5$ s	$t_{loadl} = 1,4$ s	loading time of shuttles and Lifts
$t_{unloads} = 6,5$ s	$t_{unloadl} = 1,4$ s	unloading time of shuttles and Lifts
$p_v = 0,4$ m		vertical pitch
$p_h = 0,5$ m		horizontal pitch

Figure 8 shows the analytically calculated cycle times depending on the length and height of the rack.
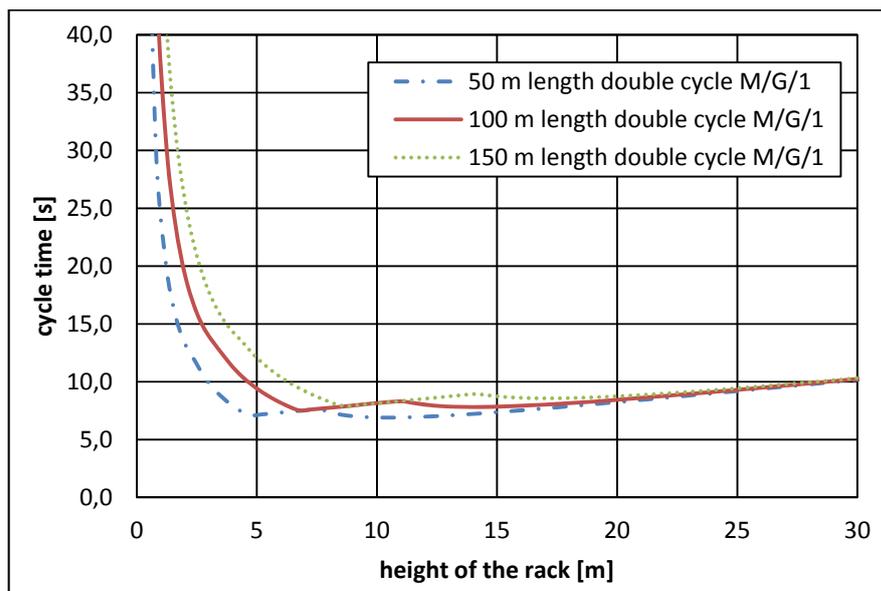
Figure 8: Cycle times t_{cycle} of a shuttle system with lifts performing single cycles and shuttles performing double cycles

Only a few shuttles operate in the left region of figure 8 with rack heights below five meters. Therefore the long travel time of the shuttles is responsible for a high cycle time. Between heights of 5 and 20 meters we observe minimal cycle times for rack lengths of

50, 100 and 150 meters. In this region the rate of utilization equals one $\rho = 1$ for the three rack lengths, which means shuttles and lifts are almost equally utilized. Here the waiting times play an important role. For rack heights higher than ≥ 20 meters only the lift performance determines the cycle time, which results in almost equal cycle times for all three rack lengths of 50, 100 and 150 meters.

4.2   Cycle times and throughputs with lifts and shuttles performing single cycles

Simulation experiments in section 5 exhibited that dual command operations of the shuttles can result in blocking effects with deadlock situations.

The following scenario leads to a dead-lock situation (Figure 9):

Level 1:   input buffer, output buffer and shuttle are occupied
              input lift shall transfer tote A to input buffer

Level 2:   input buffer, output buffer and shuttle are empty
              tote B is the next tote to be removed from storage
              shuttle waits for tote from input lift to fulfil double cycle
              input lift is caught in level 1, where it should transfer tote A to the input
buffer

These deadlock situations can be avoided by two strategies:

1. An intelligent control system with look ahead capabilities and the transfer of input loads to other levels with less traffic expectation yet to be developed
2. By exclusive single command operation of the shuttles
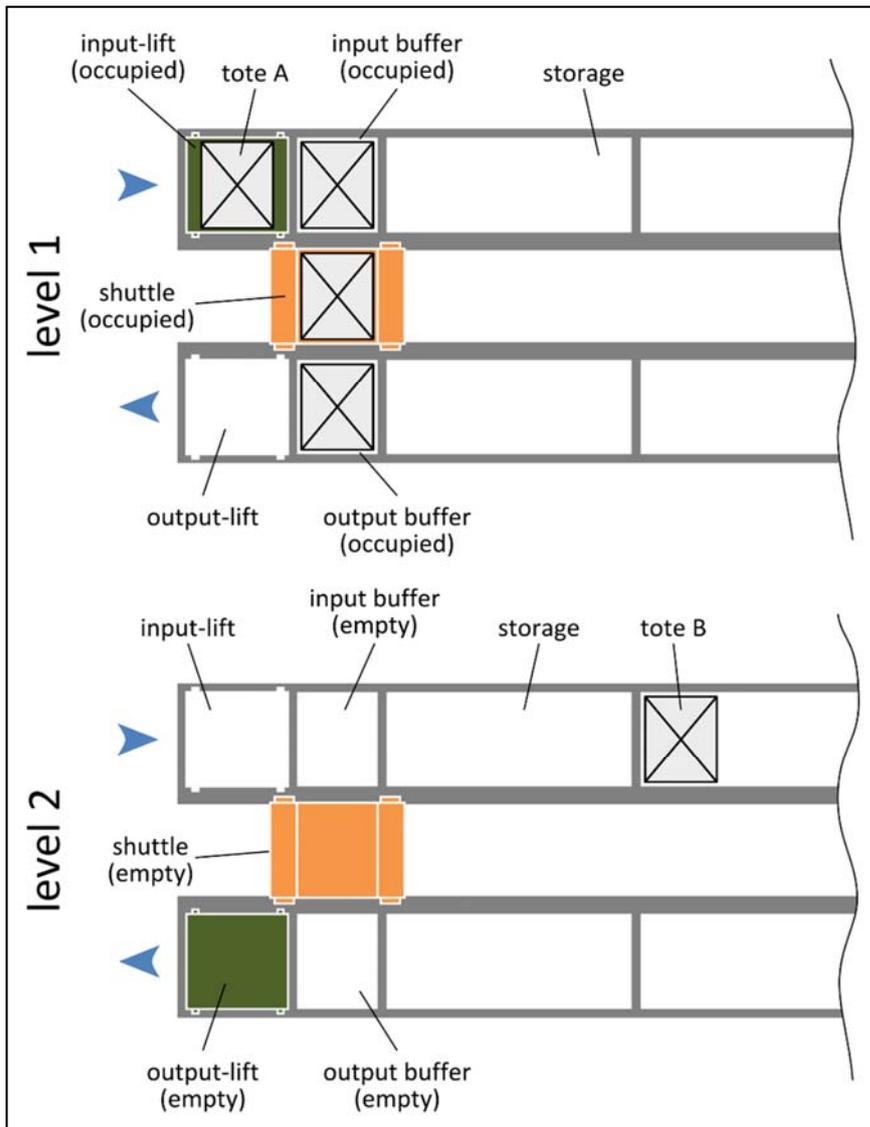
Figure 9: Deadlock situation

Single command operation of the shuttles results in the following equations with two differences when calculating the total shuttle cycle time:

1. Shuttle cycle time follows from

$$E\left(t_{shuttle}\right) := 2 \cdot \left(\frac{l_{rack}}{v_{shuttle}} + 2 \cdot \frac{v_{shuttle}}{a_{shuttle}} + t_{load\_shuttle} + t_{unload\_shuttle} \right)$$

The equation shows that all partial time elements are doubled.

2. Average waiting time in single command operation is assumed $t_{wait\_max} = 3$ sec.
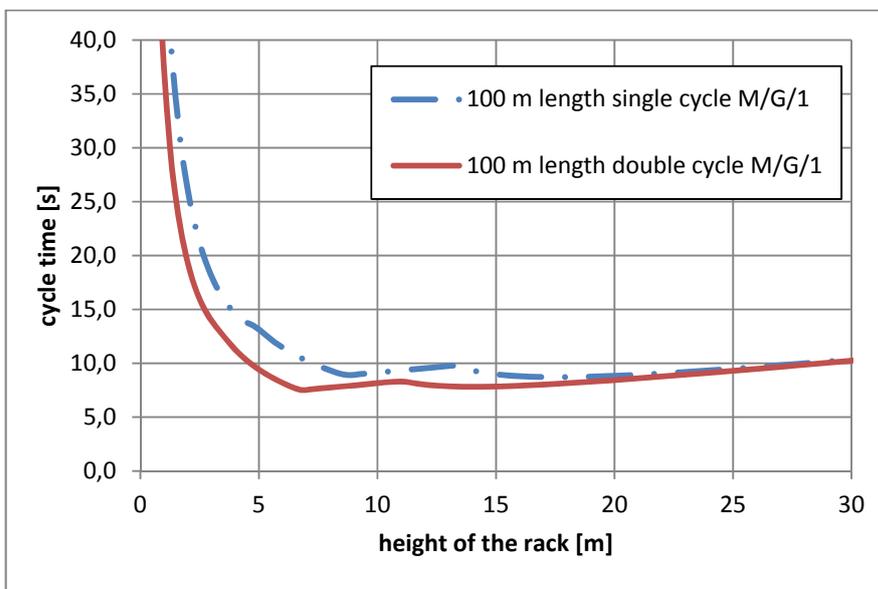
Figure 10: Comparison between the cycle time of a shuttle system with lifts and shuttles performing single cycles and a shuttle system with lifts and shuttles performing single cycles and shuttles performing double cycles
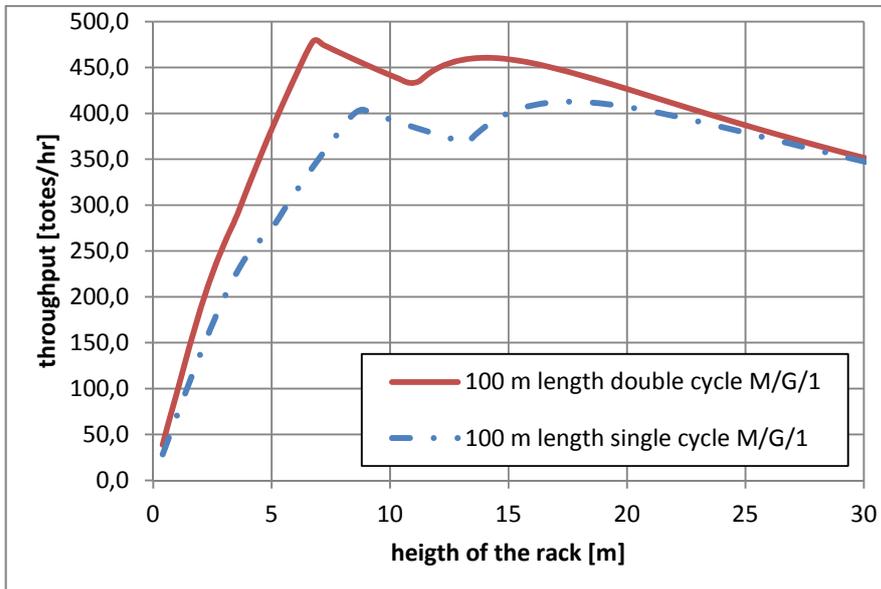
Figure 11: Comparison between the throughput of a shuttle system with lifts and shuttles performing single cycles and a shuttle system with lifts and shuttles performing single cycles and shuttles performing double cycles

Figure 10 and 11 exhibit only a small difference in SC and DC operation. A worst case scenario shows a performance loss of 100 loads/hour (Fig. 11 at 5 m to 8 m height).

5   Simulation model

To validate the results of the analytical calculation the shuttle system was also investigated by simulation with ARENA. Figure 12 shows a screenshot of a small ARENA model [6]. Tote locations in purple, buffer spaces in green. Shuttles, lifts and totes are represented by symbols.
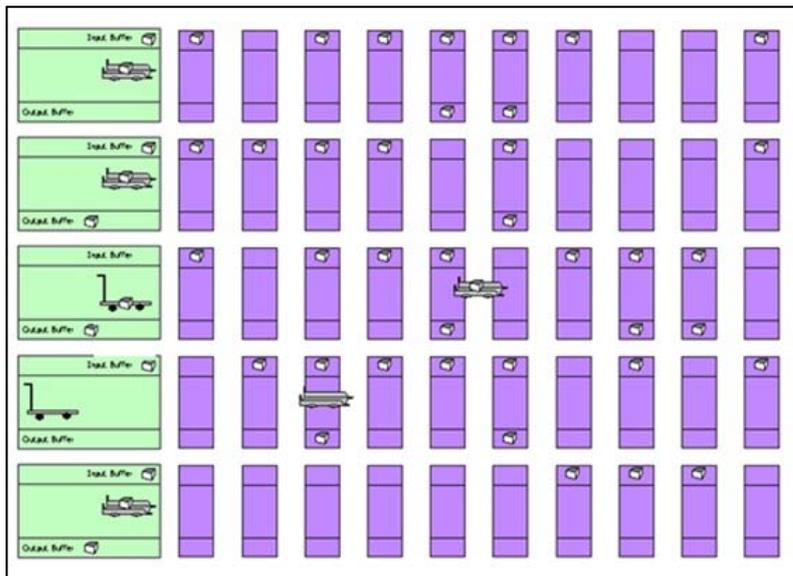
Figure 12: Screenshot of a small ARENA model with five levels and ten tote locations on each side of a level [6]

The simulation starts with an empty rack that has to be filled by the shuttles up to a filling degree of 50%. After that a random number generator defines the positions of the totes to be retrieved as well as the positions of the totes to be stored.

For the calculation of the cycle time the simulation model is set to an operating time of 10000 seconds. During this time the number of output totes n_{totes} is counted.

Figure 13 and 14 show cycle times and throughput achieved by simulation and by analytical calculation. There is a very good compliance between analytical calculation and simulation. The maximum difference between analytical calculation and simulation is about 10%. That can be seen as a proof for the correctness of both methods.
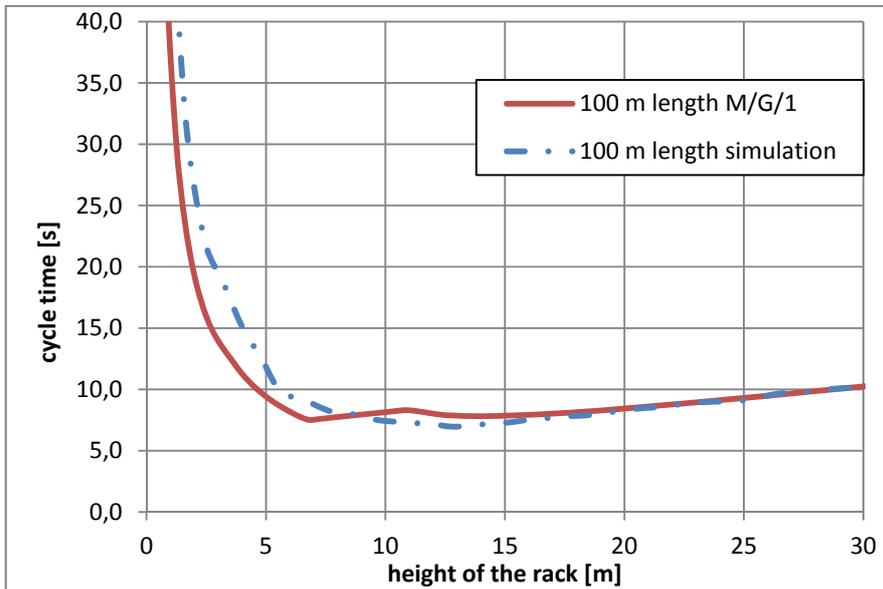
Figure 12: Cycle times t_{cycle} of a shuttle system with a rack length of 100 m achieved by analytical calculation and by simulation
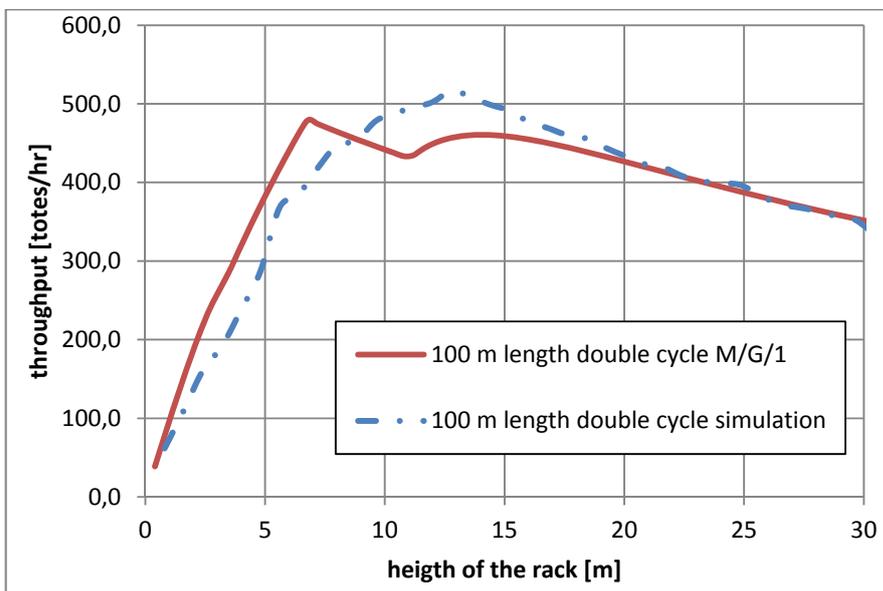
Figure 13: Throughputs of a shuttle system with a rack length of 100 m achieved by analytical calculation and by simulation

6   Comparison between shuttle throughput and AS/RS throughput

Figure 14 exhibits a comparison of the performance measures of AS/RS and shuttle systems both having the same rack geometry. Kinematic data are defined by

$v_x = 6$ m/s $v_z = 3$ m/s

$a_x = 4$ m/s^2 $a_z = 4$ m/s^2

$t_{load} = 3$ sec $t_{unload} = 3$ sec

With these data AS/RS achieve a throughput of 85 totes/hour. For the rack measures regarded here, the maximum throughput of shuttle systems lies between 450 and 500 totes/hour (Figure 14). So the throughput of shuttle systems is about five to six times higher.
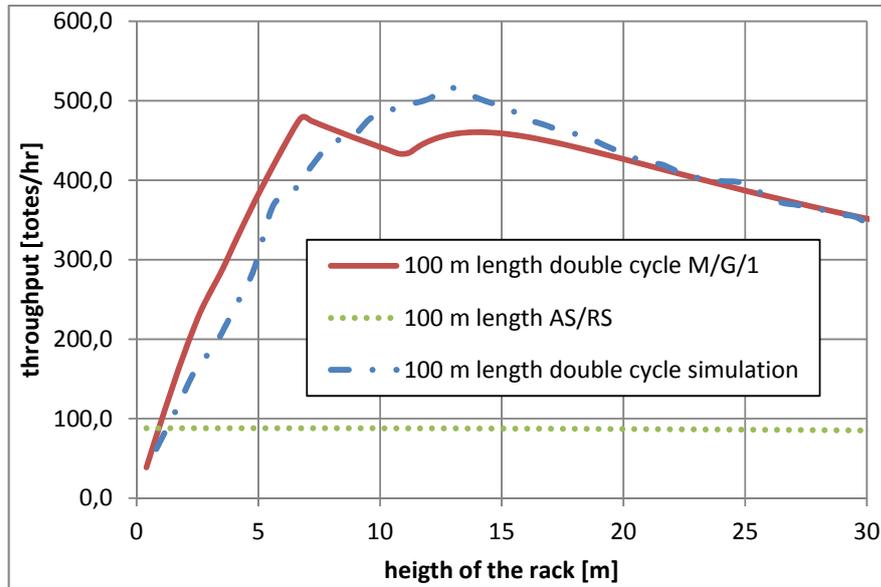
Figure 14: Throughput of shuttle system and AS/RS

7   Conclusion

This research is aimed to develop analytical methods to model throughput and cycle times of an autonomous shuttle storage system with single buffers between lifts and shuttles. Simulation results verify the correctness of the calculations. A further valuable result is the detection of blocking situations occurring in dual command operation of shuttles. These

can be avoided by intelligent control rules or by single command operation. Further research will be extended to the influence of the dwell point position, the I/O location and the number of buffers on the system performance.

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